

According to the National Curriculum for Maths, the aim is that all children, “*can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*” The combination of declarative and procedural knowledge develop pupils' conditional knowledge (ability to solve problems).

At Gawber, we know the importance of exposing children to problem solving from an early age and then maintaining this throughout their mathematical journeys. We build problem solving activities into our maths curriculum so that children's exposure to a wide variety of problems helps to develop their conditional knowledge. It's our aim to develop children's experience of problem solving as a positive approach and incorporating their energy to solve the problems through the different emotions. For example, the feeling of frustration is positive and this helps to lead them to action. We achieve this by highlighting successes, explaining how children how overcome particular feelings and through scaffolded support. Our aim to encourage children to persevere and use their resilience. A key part of our maths lessons involve oracy; the teachers encourage children to talk about maths problems with their peers.

We use the NCETM programme to deliver a high-quality maths curriculum with a focus on mastery skills. This means that children are often exposed to problems solving many times within the areas of maths they are taught. Alongside this, we also teach problem solving using techniques such as 'related questions, slow reveals and the use of representations'.

Related questions:

A question is given to the children and it is worked through as a class. Then the question is adapted slightly each time, so that the context is familiar to the children. They can build on their knowledge and see the relationship better.

 50p

 40p

Adam buys 2 drinks and 3 bananas.
How much does it cost?

 50p

 40p

Adam buys 4 drinks and 2 bananas.
How much does it cost?

 50p

 40p

Adam buys 2 drinks and 3 bananas.
He has £1. How much more money is needed?

Slow Reveals:

A question is presented to the class but it is revealed one part at a time. Initially, one piece of information is given. A blue rectangle hides another piece of information and the children try to work out what that could be. A red rectangle hides the question and children use their knowledge of the information already given to decide on possible questions.

There are 6 children at the park.



There are 6 children at the park.

There are 2 adults at the park.



There are 6 children at the park.

There are 2 adults at the park.

How many people at the park?

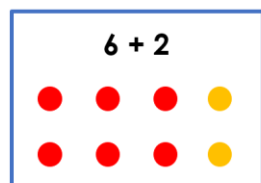
Representations:

Once a question has been asked, children match the correct representation to the procedure. This could include counters, bar models, symbols etc or even children creating their own representations. The representations are colour coded, so when using mass participation it is clear if children have understood straight away.

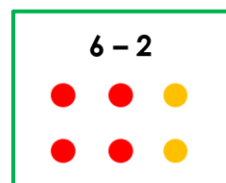
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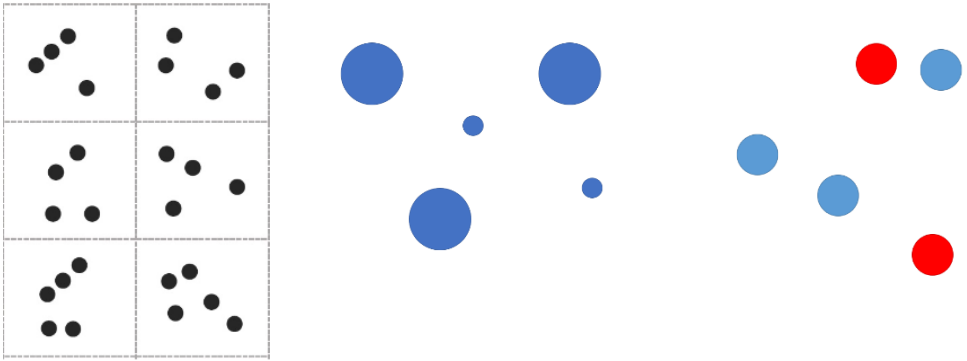
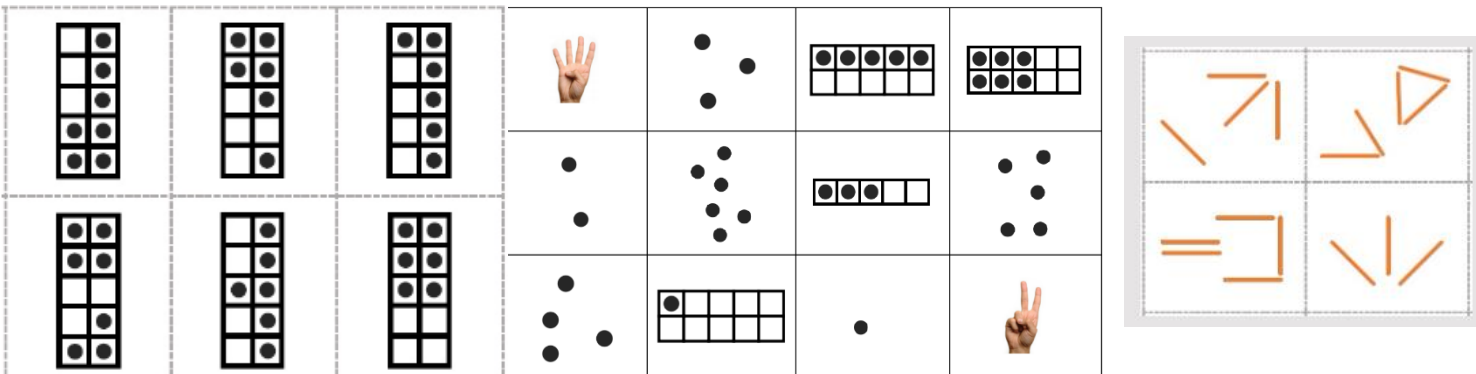
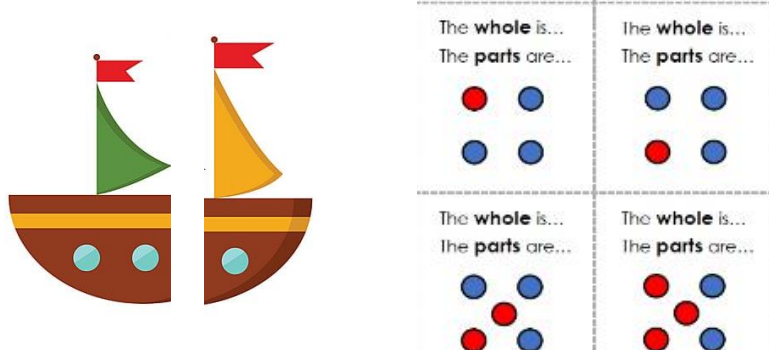
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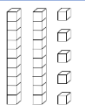
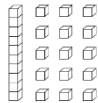
Which Picture?



The next pages show the progression in problem solving across our school. These are some of the examples that can be used in each year group.

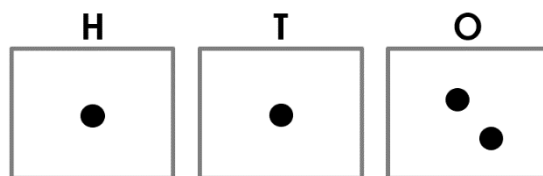
	How many dots? Focus on subitising. Variation is used by differing the size and colours.	
EYFS	How many dots? Using 5 and 10 frames. Children are shown 5 and 10 frames and become familiar with the manipulative and representation. Play number bingo and children are again exposed to seeing numbers represented in different ways. Then repeat using lines.	
	Part-whole games Start by using familiar pictures and introduce the part, part, whole vocabulary. Move on to using dots and numbers.	

Year 1	Recognising Doubles Using tens frames, children can create doubles using manipulatives. Then progress to using representations, then numbers.	Double 6 is 12												
	Which picture? Children look at the word problem and select the colour of the representation that is correct. Then progress on to drawing their own picture to represent the problem. Children are then given a representation and a number sentence and they create their own word problem.	Which Picture? Kate has 3 apples. Tom has 2 apples. How many apples in total? 3 + 2 Which Picture? 3 - 2 <table><tr><th>Question:</th><th>Picture:</th><th>Number Sentence:</th></tr><tr><td>There are 5 apples. 3 of the apples are red. How many apples are green?</td><td></td><td>5 + 3 OR 5 - 3</td></tr></table> <table><tr><th>Question:</th><th>Picture:</th><th>Number Sentence:</th></tr><tr><td></td><td> </td><td>5 + 2 OR 5 - 2 </td></tr></table>	Question:	Picture:	Number Sentence:	There are 5 apples. 3 of the apples are red. How many apples are green?		5 + 3 OR 5 - 3	Question:	Picture:	Number Sentence:			5 + 2 OR 5 - 2
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Slow Reveal Children are shown the initial piece of information and the question. They have to work out what piece of information is hiding behind the blue box.	There were 8 people on the bus. information How many people left on the bus?													

	Slow Reveal Then children are given two pieces of information and they decide what the questions could be.	There were 8 frogs in the pond. There were 2 frogs on the lily pad. question
Year 2	Slow Reveal Children start with one piece of information.	In total, Max and Dan have 6 sweets.  
	They have to think of possible options for the second piece of information.	In total, Max and Dan have 6 sweets. Max has 2 sweets. 
	Discuss possible questions that could be asked using the two pieces of information.	In total, Max and Dan have 6 sweets. Max has 2 sweets. How many sweets does Dan have?
	Look at the representations and children select which option is correct.	In total, Max and Dan have 6 sweets. Max has 2 sweets. How many sweets does Dan have? $6 + 2$  Which Picture? $6 - 2$ 
	Number Children are exposed to seeing numbers presented in many different ways.	Building 2-Digit Numbers Part 1 Is it 25? ✓ or ✗  $5 + 20$ 5 tens 2 ones 

	Number lines As children's knowledge of number develops, the types of number lines they will see will become more challenging.	Which numbers are at the arrows?
Year 2	Pattern Seeking What comes next? Start with numbers, then shapes etc.	What next? 0 0 1 1 2 2 3 □ △ □ △ □ △ □ □ □ □ □ □ ○ ○ ○ ○ ◇ 0 0 0 0 0 0 0 Match a number pattern to a shape pattern. 4 4 5 5 6 6 4 6 4 6 4 6 4 4 5 5 4 4 ⊕ ⊕ △ △ ⊕ ⊕ ⊕ ⊕ △ △ ☆ ☆ ⊕ ☆ ⊕ ☆ ⊕ ☆
	Dots on a grid Children use their maths knowledge to solve the missing numbers. Then progress on to some numbers and some counters missing.	
	Fractions Children begin to understand the importance of equal parts.	As a fraction of the shape, is the coloured part... Less than $\frac{1}{4}$ $\frac{1}{4}$ More than $\frac{1}{4}$

Place value
Use counters to represent numbers.



103

112

Number lines
Children apply their knowledge of number and place value. Ask them to locate each number first. Then explore difference etc.



Smallest to largest: blue (32→40), purple (84→100), green (40→59), orange (59→84), red (0→32).

Pattern and sequences
Children begin to notice patterns in times tables. E.g. even numbers.

$1 \times \square = \square$		
$2 \times \square = \square$	1	
$3 \times \square = \square$		8
$4 \times \square = \square$		
$5 \times \square = \square$		0
$6 \times \square = \square$		
$7 \times \square = \square$		2
$8 \times \square = \square$		
$9 \times \square = \square$		4
$10 \times \square = \square$		

First 2-digit number here: true for 5→9 times tables.

Must be even number: when multiplied by 5 even numbers end in 0.

To have these ones values, this must be the 6 × table.

Number Sentences
Starting with more simple numbers initially. Children use their knowledge of commutativity.

Number sentences using:

$+$ $=$ 7 4 11

$7 + 4 = 11$ $11 = 7 + 4$

$4 + 7 = 11$ $11 = 4 + 7$

Number sentences using:

$-$ $=$ 7 4 11

$11 - 7 = 4$ $7 = 11 - 4$

$11 - 4 = 7$ $4 = 11 - 7$

Number sentences using:

$+$ $=$ 7 4 3

$4 + 3 = 7$ $7 = 4 + 3$

$3 + 4 = 7$ $7 = 3 + 4$

Number sentences using:

$-$ $=$ 7 4 3

$7 - 4 = 3$ $4 = 7 - 3$

$7 - 3 = 4$ $3 = 7 - 4$

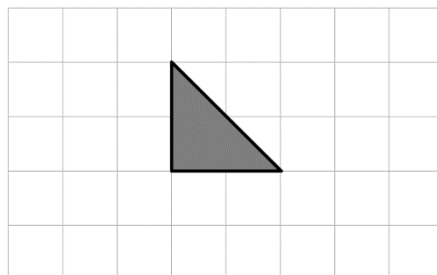
Bar models

Children discuss a context that the picture represents. What can be concluded from the bar model? E.g. John has £4.

£12		
John	Friend 1	Friend 2

Fractions

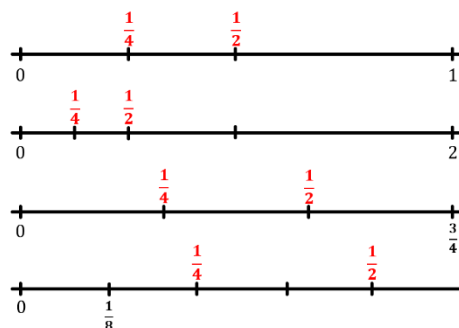
Children will now be more familiar with fractions. Ask questions such as... if this was $\frac{1}{4}$ of the whole, what could the whole look like?



Fractions on a number line

Begin by asking children to plot fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$

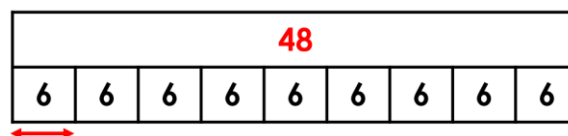
Link to word problems.



Fractions of amounts

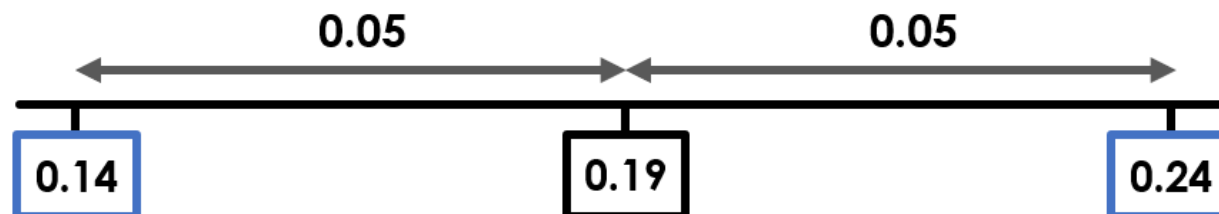
Children begin to see how to solve fractions of amounts. Vary the number of parts (denominator) and allow them to explore what happens.

Example answer 3: $\frac{1}{9}$ of 54 = 6



Decimal number lines

Children are given one number and they have to use the amounts increased or decreased by in order to work out the missing numbers.



More than, less than and equal

Children work in pairs to put the digits in the correct place. Encourage discussions and reasons why or why not.

4, 5, 6, 7, 8

$$10 - 8 < \square - \square$$

$$20 > \square \times 3$$

$$\square + 4 = 15 - \square$$

Missing digits

Children begin by saying what they know. E.g. start with the ones column first. 5 add something must equal a number ending in 2 e.g. 12.

$$\begin{array}{r} \square 8 \square \\ + 3 \square 5 \\ \hline 1052 \end{array}$$

$$\begin{array}{r} 7 \\ 6 \square 2 \\ - \square 3 \square \\ \hline 243 \end{array}$$

More than one answer problems
Children begin by recapping what they know e.g. define sum and difference. Children explore using different numbers and adjust depending on their answer.

Two numbers: sum = 9, difference = 4

6 and 3  **sum = 9 ✓**  **difference = 3 ✗**

6.5 and 2.5  **sum = 9 ✓**  **difference = 4 ✓**

7 and 2  **sum = 9 ✓**  **difference = 5 ✗**

532

1 2 3 4 5 6 7 8 9

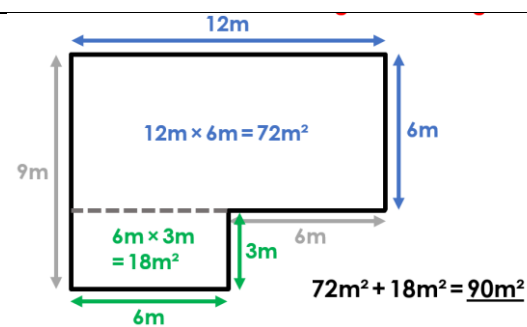
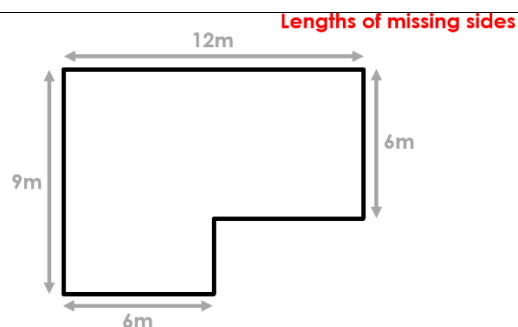
Without calculating we know...

Factors and multiples

Children start by using the facts they know e.g. 600 is a multiple of 3, 540 is too, so 3 isn't a factor.

Area and perimeter

Using a compound shape, ask children to explain what they already know. How can we work out the sides that have no measurements? How do you split the shape? Etc.



Co-ordinates

Recap 'along the corridor, up the stairs'. What does (7,2) actually mean? Point to another location that would begin with (7,...) How can we begin to work out the co-ordinates for A? What do we already know that can help us?

